THE OPTIMUM DESIGN OF SEMICONDUCTOR LIQUID-FLOW COOLING MECHANISMS

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We have found the distribution for the density of the electric current in a thermoelectric battery to ensure the greatest energy efficiency under conditions of liquid-flow streamlining. The optimum current density and the nature of its change along the battery are functions of the temperature conditions, the cooling (refrigerating) capacity, the parameters of the liquid flows, the geometric dimensions of the battery, and the physical characteristics of the thermoelectric elements.

Semiconductor thermoelectric liquid-flow cooling and heating devices operate under conditions in which the thermoelectric elements—positioned along the flow—operate under various temperature conditions. Earlier [1, 2] we proposed methods of designing batteries to ensure maximum energy efficiency, with consideration of the limitations imposed on battery dimensions; it was assumed in this case that the battery is made up of like elements and that the supply current is identical for each. The efficiency of the thermal battery can be improved by using a variety of supply-current densities for the thermoelectric elements situated along the liquid flow; this is achieved most simply by using elements of various dimensions [3]. It is assumed in [3], as well as in later publications [4, 5] devoted to the design of cooling and heating mechanisms with maximum energy efficiency that the current passing through each thermoelectric element must correspond to the condition of a maximum coefficient of energy efficiency. All of the battery elements are usually series-connected into the electric circuit; this forces us to change the dimensions of the elements in accordance with the temperature distribution along the battery. The magnitude of the current achieving the maximum value for the conversion factor [6] is given by

$$I_0' = \frac{\alpha s \Delta T}{\rho d\left[\sqrt{1+0.5z(T_1+T_2)}-1\right]}. \qquad (1)$$

When the current in each of the elements is subject to relationship (1), the entire battery functions with maximum energy efficiency. However, if a current $I = I_0$ passes through each of the battery elements, the cooling capacity of the elements is less than the limit; therefore, to attain the specified capacity we will have to increase the total number of elements or the size of the battery. This becomes particularly evident if the temperature difference $\Delta T = T_2 - T_1$ is not very great. In practice, the difference $T_2 - T_1$ at the initial segment of the battery in a number of cases may be equal to or less than zero. Here, relationship (1) loses significance and the method proposed in [4, 5] cannot be used at all.

Let us consider the problem of finding the optimum distribution for the dimensions of the battery elements along the liquid flow (or what is the same, the distribution of the current density) to ensure a specific cooling capacity for the batteries with maximum energy efficiency, considering the specified area limitations. The water equivalence of the flows and their initial temperatures are assumed to be known.

To simplify the calculations, we will schematize the actual conditions slightly. The coefficients for the transfer of heat between the junctions and the liquid flows are assumed to be rather large and we neglect the temperature difference between the battery surface and the liquid flows. In practical terms, as follows, for example, from [2, 7], such conditions prevail when Bi > 15–20. Moreover, we assume that the water equivalent of the flow streamlining the hot-junction side is so great that we can neglect the temperature differences at the inlet and at the outlet. Such conditions are characteristic of a number of installations employing intensive hot-junction cooling; these generally are found in air and water conditioners.

In the case under consideration the system of differential equations which, under steady-state conditions, characterize the change in the heat content of the parallel liquid flows streamlining the junctions of the thermal battery have the form

$$-\frac{W_1}{p}\frac{dT_1}{dx} = \alpha jT_1 - \frac{1}{2}j^2\rho d - \frac{\lambda}{d}(T_2 - T_1), \qquad (2)$$

$$\frac{W_2}{p}\frac{dT_2}{dx} = \alpha jT_2 + \frac{1}{2}j^2\rho d - \frac{\lambda}{d}(T_2 - T_1), \quad 0 \leqslant x \leqslant l \qquad (3)$$

The boundary conditions $T_1(0) = T_1^{(0)}$, and $T_2(0) = T_2^{(0)}$. Since the total cooling capacity of the battery is specified, the temperature of the cooling flow at the outlet is correspondingly specified as $T_1(l) = T_1^{(l)}$ These conditions pertain to a single-pass flow; however, in this case of a constant $T_2(x)$, the results of the calculation are independent of the respective directions of liquid motion.

Equations (2) and (3) can be presented in dimensionless form, introducing the same variables and dimensionless complexes as in reference [1]:

$$-\frac{1}{R_1}\frac{d\tau_1}{d\xi} = \frac{dq_1}{d\xi} = (1+U)\tau_1 - \tau_2 - \frac{U^2}{2}, \qquad (4)$$

$$\frac{1}{R_2}\frac{d\tau_2}{d\xi} = \frac{dq_2}{d\xi} = -(1-U)\tau_2 + \tau_1 + \frac{U^2}{2},$$

$$0 \leqslant \xi \leqslant 1, \ \tau_1(0) = \tau_1^{(0)}, \ \tau_2 = \tau_2^{(0)}, \ \tau_1(1) = \tau_1^{(1)},$$
$$q_1(0) = 0, \ q_1(1) = q_1^{(1)}, \ q_2(0) = 0. \qquad (5)$$

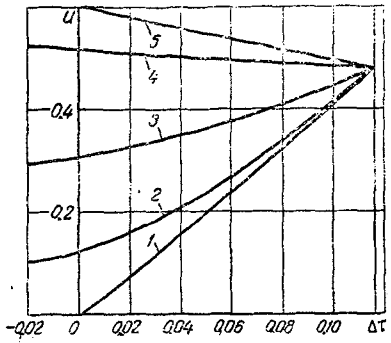


Fig. 1. Current-density distribution over battery versus temperature of cold junctions ($\tau_2$ = 0.6): 1) C = 0; 2) C = 0.01; 3) C = 0.1; 4) C = 1; 5) $C \to \infty$.

We can now formulate the problem of calculating the parameters for a battery with maximum energy efficiency in the following manner. We have to determine a function $U(\xi)$, such that the solution of system (4) and (5), satisfying the corresponding boundary conditions, will yield the least value for $q_2(1)$ (it is clear that the smallest value of $q_2(1)$ will correspond to the maximum of the cooling coefficient $\varepsilon$).

The formulated variational problem—with the conditional extremum—is the Mayer problem [8]. This problem can be solved by classical methods. The most common approach is that based on the maximum principle [9], which permits us to impose limitations on U, for example, of the form $U \le U_0$.

For the solution let us use the maximum principle in that of its interpretations which pertains to problems with fixed time [9] (in our case, instead of the time interval it is the length of the segment along the dimensionless coordinate $\xi$ that is fixed). Then the auxiliary function

$$H = \Psi_1[-(1+U)\tau_1 + \tau_2 + 0.5U^2] + \Psi_2[-(1+U)\tau_2 + \tau_1 + 0.5U^2]. \tag{6}$$

Since $\tau_2 = \tau_2^{(0)}$ = const, we have $\Psi_2 = -1$. Moreover,

$$H = 2C = \text{const} \geqslant 0, \quad \frac{\partial H}{\partial U} = 0. \tag{7}$$

Eliminating $\Psi_1$ from system (7), we find that

$$U = \frac{\Delta\tau + 2C\tau_1(\Delta\tau)^{-1}}{\{[1 - C(\Delta\tau)^{-1}]^2 + 0.5[1 + 2C\tau_1(\Delta\tau)^{-2}](\tau_1+\tau_2)\}^{\frac{1}{2}} + C(\Delta\tau)^{-1} - 1}. \tag{8}$$

The constant C as a function of the initial data of the problem can assume values from zero to infinity. In a specific case, to determine C, we have to integrate Eq. (4), substituting the expression in (8) for U in this case. Because the corresponding integral can be evaluated only numerically, we are unable, in general form, to derive the analytical relationship between C and $\tau_1^{(0)}$, $\tau_2^{(0)}$, $\tau_1^{(1)}$, and $R_1$.

As is easily seen, the limit values of C correspond to conditions of a maximum cooling coefficient and maximum cooling capacity for all of the battery elements. Indeed when C = 0

$$U = U_1 = \frac{\Delta\tau}{[1 + 0.5(\tau_1+\tau_2)]^{\frac{1}{2}} - 1}$$

and as $C \to \infty$, $U = U_2 = \tau_1$. For all the intermediate values of C, $U_1 < U < U_2$.

Regardless of the magnitude of C, the right-hand member of Eq. (4) always vanishes when the temperature difference $\tau_2 - \tau_1$ across the battery becomes equal to $0.5\,\tau_1^2$ or $\tau_1 = b - 1$. It is natural that the limit temperature difference across the battery streamlined by flows of a heat carrier [coolant] cannot exceed the maximum difference across the thermoelectric element in the absence of a heat load which, as is well known [6], is equal to $0.5\,\tau_1^2$.

The basic parameters for the thermoelectric batteries with a variable current density were calculated with the aid of a computer. For the various values of the constant C we found the functions $U(\tau_1)$ and $U(\xi)$, the $R_1$ numbers, and the cooling coefficients $\varepsilon$. The calculations were carried out for a number of values of the initial temperature $\tau_1^{(0)}$ and for temperature differences of $\Delta\tau_1 = \tau_1^{(0)} - \tau_1^{(1)}$ in the flow. The values of $R_1$ and $\varepsilon$ were determined by numerical integration of Eqs. (4) and (5) with consideration of (8), and here

$$R_1 = \int_{\tau_1^{(1)}}^{\tau_1^{(0)}} \frac{d\tau_1}{(1+U)\tau_1 - \tau_2 - 0.5U^2}, \tag{9}$$

$$\frac{1}{\varepsilon} = \frac{q_2(1) - q_1(1)}{q_1(1)} = \frac{1}{\Delta\tau_1}\int_{\tau_1^{(1)}}^{\tau_1^{(0)}} \frac{U(U + \tau_2 - \tau_1)\,d\tau_1}{(1+U)\tau_1 - \tau_2 - 0.5U^2}. \tag{10}$$

The temperature variation along the battery can be found by integrating (4) with consideration of (8) and (9). Data on the change in current density $U(\xi)$ along the length of the battery are obtained on substitution of $\tau_1(\xi)$ into (8).

Figure 1 shows the curves of the change in current density along the battery as a function of the junction temperatures for various values of the parameter C. When $\tau_2$ = 0.6, the limit difference across the battery is $\Delta\tau = \tau_2 - \tau_1 = b - 1 = 0.117$; the value of U at this point is independent of C and equal to $\tau_1$, i.e., 0.483. The curves in Fig. 2 show the function $U(\xi)$ for one variation of the temperature conditions (the values of C are the same as in Fig. 1).

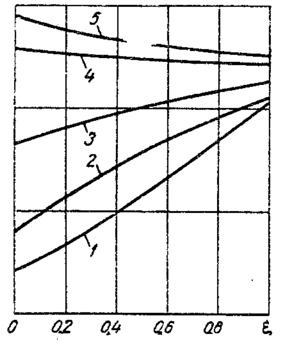


Fig. 2. Current-density distribution along battery length: ($\tau_2 = 0.6$, $\tau_1^{(0)} = 0.58$, $\tau_1^{(1)} = 0.5$).

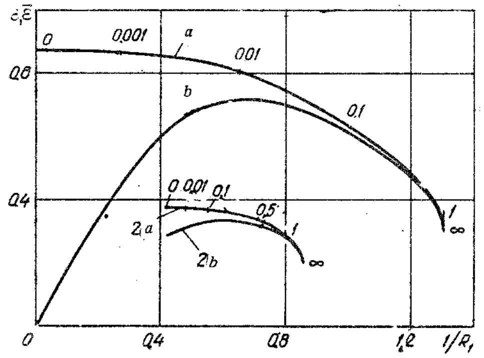


Fig. 3. Cooling factor versus parameter $R_1(\tau_2 = 0.6)$: 1) $\tau_1^{(0)} = 0.6$; $\Delta\tau_1 = 0.08$; 2) 0.58 and 0.08.

We obtained comparative data on the efficiencies of the batteries operating under the conditions $U(\xi) = \bar{U}$ = const on the basis of the relationships presented in [1,2]. To determine $\bar{U}$ for specified $\Delta\tau_1$, $\tau_1^{(0)}$, $\tau_2$, and $R_1$ we have to solve the transcendental equation

$$\Delta\tau_1 = [\tau_1^{(0)} - (\tau_2 + 0.5\bar{U}^2)(1+\bar{U})^{-1}] \times \{1 - \exp[-R_1(1+\bar{U})]\}. \tag{11}$$

Here the cooling coefficient

$$\bar{\varepsilon} = \frac{(1+\bar{U})\Delta\tau_1}{\bar{U}[(\tau_2+1+0.5\bar{U})\bar{U}R_1 - \Delta\tau_1]}. \tag{12}$$

Figure 3—as an example—shows two pairs of curves which define the cooling coefficient as a function of the parameter $R_1$ for a specified temperature difference $\Delta\tau_1$. The curves identified by the letter *a* describe batteries with an optimum distribution of the current density over the area, while the curves identified with the letter b refer to the batteries with U = const. The resulting data show that the battery with the optimum distribution U(x) exhibits the greatest energy efficiency. A relative increase of 20–30% over the cooling coefficient of a battery with U = const is attained. The $\varepsilon = f(R)$ data are derived for the 0 to $\infty$ interval of C. There is a monotonic increase in $\varepsilon(R)$ in the corresponding range of R, and $\bar{\varepsilon}$ exhibits a maximum. When $C \to \infty$, $U = \tau_1$, i.e., the battery is operating under conditions of maximum cooling capacity. For specified values of $\tau_1^{(0)}$, $\tau_1^{(1)}$, and $\tau_2$, such a regime assures a minimum value for $R_1$. In accordance with (9) and (10)

$$R_1 = \frac{1}{b}\ln\frac{(1+b+\tau_1^{(1)})(1-b+\tau_1^{(0)})}{(1-b+\tau_1^{(1)})(1+b+\tau_1^{(0)})}, \tag{13}$$

$$\frac{1}{\varepsilon} = \frac{\tau_2}{b\Delta\tau_1}\left[(b-1)\ln\frac{1-b+\tau_1^{(0)}}{1-b+\tau_1^{(1)}} + (b-1)\ln\frac{1+b+\tau_1^{(0)}}{1+b+\tau_1^{(1)}}\right]. \tag{14}$$

The temperature distribution along the battery, defining the optimum change in current density,

$$\tau_1 = b - 1 + 2b[A\exp(bR_1\xi) - 1]^{-1}. \tag{15}$$

Here

$$A = \frac{1+b+\tau_1^{(0)}}{1-b+\tau_1^{(0)}}.$$

A thermal battery with a current density of $U(\xi)$ = const can no longer provide the same heating capacity as a battery with a variable U when $U = \tau_1$ and when the values of $R_1$ are equal. However, these maximum magnitudes for the cooling capacity differ only slightly, within the limits of tenths of a percent. Thus, the use of elements with variable dimensions is virtually without point in the case of a battery with a relatively small parameter $R_1$ (a small ratio of area to the water equivalent of the flow), where this battery is operating in a regime close to that of maximum cooling capacity. The greatest gain in efficiency is achieved through the use of a battery with a relatively large $R_1$, in which case it is possible to achieve conditions for each of the elements that are close to the regime of the maximum cooling coefficient.

If we know the current-density function $U(\xi)$ along the battery, we can determine the number of elements n per unit area for a specified current I:

$$n = \frac{j}{I} = \frac{U\lambda}{I\alpha d}.$$

For practical purposes, in the place of a continuous change in the dimensions of the elements we can naturally use a stepwise variation in element dimensions, limiting ourselves to two or three steps.

This article describes a method for the optimization of thermoelectric cooling devices; an analogous approach will enable us to raise the efficiency of heating installations and of thermoelectric generators.

NOTATION

$T_{1,2}$ is the temperature in the fluid flow; $T_{1,2}^{(0)}$ is the temperature of the heat-transfer agent at the entrance; $\Delta T_{1,2}$ is the temperature difference along battery length; $\Delta T = T_2 - T_1$ is the temperature drop across the battery thickness; S, d, $l$, and p are the area, thickness, length, and width of the battery. $\alpha$, $\lambda$, and $\rho$ are the reduced coefficients of emf thermal conductivity, specific resistance; $z = \alpha^2/\rho\lambda$; I and j are the current intensity and density; s is the cross section of the element; n is the number of elements per unit area; $W_{1,2}$ is the water equivalent of the flow; x is the coordinate along the flow; $\xi = x/l$ is the dimensionless coordinate; $\tau_{1,2} = zT_{1,2}$ is the dimensionless temperature; $R = S\lambda/Wd$; $U = \alpha jd/\lambda$ is the dimensionless current density; $q = \Delta\tau_{1,2}/R_{1,2}$ is the dimensionless cooling and heating capacity; $\varepsilon = q_1/(q_2 - q_1)$ is the cooling factor; $b = \sqrt{2\tau_2 + 1}$ symbols: 1 refers to the cooling flow; 2 refers to the heating flow; the overscore denotes parameters of the battery with U(x) = const.

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